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INFLUENCE OF THE CURVATURE OF AN AIRFOIL ON THE STRUCTURE OF TRANSONIC FLOW

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A study is made of the dependence of the pattern of transonic flow about an airfoil on the change in its shape and in the parameters of the incoming air flow. Consideration is given to a family of airfoils in a channel modeling the working section of a wind tunnel. The formation and propagation of weak shock waves in local supersonic regions near the central part of the airfoil has been investigated numerically. The finite-difference method based on a substantially nonoscillatory scheme of second order of accuracy has been employed to solve the Euler equations describing nonviscous gas flow. It has been shown that the sensitivity of the flow pattern to a change in the Mach number of the free flow substantially increases with decrease in the curvature of the airfoil.

At high subsonic flying speeds typical of civil and transport aviation, local zones of supersonic flow velocity are formed near the wing. There are methods of designing airfoils and wings of a "shock-free" shape that ensure smooth acceleration and subsequent deceleration of the flow back to subsonic velocities. However, any small change in the shape of a shock-free airfoil or in the design Mach number of the incoming flow for which this airfoil is designed leads to the appearance of considerable gradients in the flow and to the formation of shock waves. The occurrence of the latter is often accompanied by a high sensitivity of the flow to a change in the velocity of the free flow.

G. Cosentino [1] has investigated the example of transonic flow near an airfoil for which shock-free flow was implemented for $M_d = 0.7725$. As the Mach number decreased to 0.76, i.e., by 1.6%, a shock wave was formed near the central part of the airfoil and the supersonic region was split into two parts.

A high sensitivity of transonic flow to a variation in the parameters of the incoming flow has also been observed in [2], where an X63T18S shock-free airfoil was designed for $M_d = 0.7815$ and the lift coefficient $C_L = 0.524$. Under design conditions, flow near the airfoil was smooth and the shape of the supersonic region was convex. At the same time, a strong sensitivity of flow to small changes in the angle of attack, the Mach number of the incoming flow M_∞, and the shape of the airfoil was detected. In particular, the calculation results showed a wavy shape of the boundary of the supersonic region for $M_\infty = 0.7660 < M_d$, i.e., with a decrease of 1.9% in the Mach number.

The results of flow about a shock-free airfoil designed for $M_d = 0.64$ and an angle of attack of 2^o are presented in [3]. The decrease in the Mach number from 0.64 to 0.62 also led to the formation of concave portions in the boundary of the supersonic region and substantially changed the flow pattern.

Transonic flow with a local supersonic region in a channel with a shock-free airfoil of thickness 0.0932 designed for $M_d = 0.675$ has been investigated in [4]; it has been shown that the supersonic region is split into two individual parts as the Mach number decreases from 0.675 to 0.671 at the outlet from the channel. Both the first supersonic zone and the second one were closed by shock waves bent upstream and perpendicular to the airfoil at the points of intersection of it.

The nature of the above sensitivity of steady-state transonic flow to a change in the boundary conditions is not quite known at present. Meanwhile, it can lead to a considerable change in the lift and the total resistance of the airfoil. In particular, the presence of atmospheric inhomogeneities is capable of producing substantial dynamic loads under conditions of actual flight. Furthermore, the temperature deformations occurring because of convective and radiative heat exchange with the environment are able to appreciably change the geometry of the wing, leading to a deviation of its aerodynamic quality from the design value.

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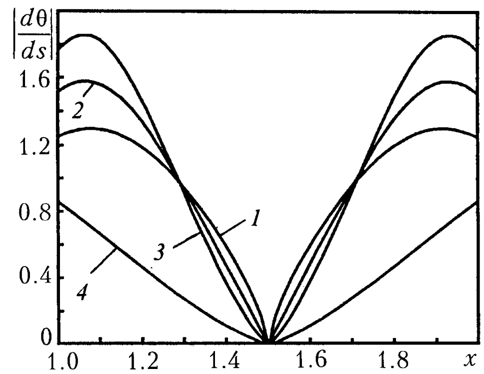

Fig. 1. Curvature of the airfoil (1) for different values of the parameter m and thickness y_{max} (θ is the angle made by the tangen to the airfoil with the x axis and s is length of the arc along the airfoil): 1) m = 2.7, 2) 3.0, 3) 3.3 (all for y_{max} = 0.1) and 4) 3.3 for y_{max} = 0.03.

This work seeks to numerically model two-dimensional transonic flow about a family of simply shaped airfoils and to analyze the dependence of these patterns on the changes in the Mach number of the free flow. Understanding the reasons why transonic flow is sensitive to a change in the boundary conditions can contribute to the development of the concepts of control over transonic flow (which are aimed at attaining a higher aerodynamic quality of the wing and a higher stability of flight) and to further investigations into the problems of nonuniqueness of numerical solutions obtained for a number of airfoils [5, 6].

Formulation of the Problem. We consider two-dimensional nonviscous flow of a compressible gas in the channel $0 < x < 3$, $0 < y < 1$ of unit width with parallel walls and a small step modeling the airfoil and located on the lower wall of the channel for $1 < x < 2$. To analyze the sensitivity of transonic flow to changes in the Mach number at the outlet from the channel we employ the family of airfoils specified by the simple analytical dependence of [7]:

$$y(x) = y_{max}(1 - |2x - 3|^m) \quad \text{for} \quad 1 < x < 2 . \tag{1}$$

Obviously, the curvature of the central part of the airfoil (1) dependent on the derivative $y''(x)$ decreases with increase in the parameter m (Fig. 1). If $m > 2$, then $y''(1.5) = 0$; therefore, at the point of the airfoil with a coordinate of x = 1.5 the curvature vanishes.

As the boundary conditions for the system of continuity, momentum, and energy equations for the gas density $\rho(x, y, t)$, the velocity components $u(x, y, t)$ and $v(x, y, t)$, and the internal energy $e(x, y, t)$ we specify the following conditions;

(a) the nonflow conditions on channel walls;

(b) static pressure in the flow p_{out} at the outlet from the channel $x = 3$;

(c) the condition of parallelism of the flow to channel walls, i.e., the zero vertical component of the velocity $v(0, y, t) = 0$ at the inlet to the channel $x = 0$; furthermore, we specify the values of the entropy p/ρ^γ and the enthalpy $\frac{u^2 + v^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}$.

The necessity of specifying three parameters at the inlet to the channel and one parameter at the outlet in solving the Euler equations for two-dimensional gas flow has been substantiated in a number of works [8, 9]. In the case of unsteady flow one must additionally specify the values of the parameters of the gas at the initial instant of time $t = 0$.

For a more convenient characteristic of the initial data, instead of the static pressure at the outlet p_{out}, below we employ the M'_{out} number obtained from the isentropic relation $p_0/p_{out} = [1 + (\gamma - 1)M^2/2]^{\gamma/(\gamma-1)}$, in which the total pressure p_0 is determined by the specified values of the entropy and the enthalpy at the inlet to the channel. As is well known, the quantity M'_{out} coincides with the real Mach number in the upper part of the outlet cross section of the channel (this part is reached by the streamlines located above the local supersonic region). At the same time, the

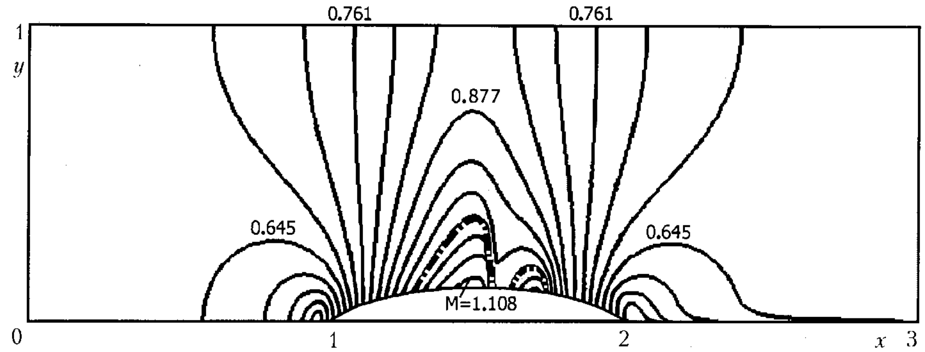

Fig. 2. Lines of constant values of the Mach number in a channel for y_{max} = 0.1, m = 2.7, and M'_{out} = 0.666.

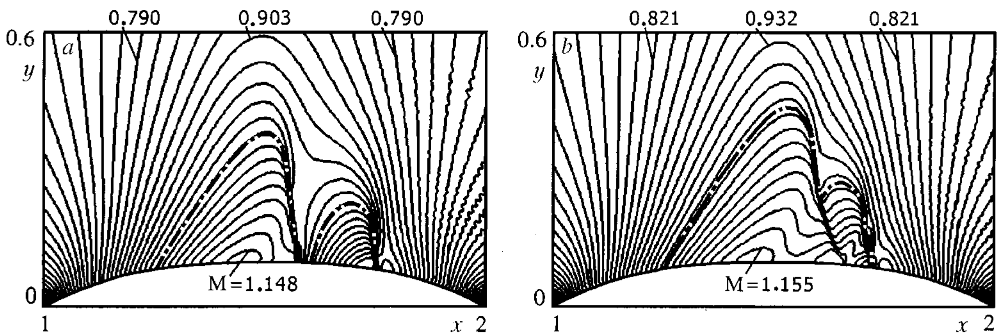

Fig. 3. Fragment of transonic flow near the airfoil. The lines of constant values of the Mach number for y_{max} = 0.1 and m = 2.7: a) M'_{out} = 0.666; b) M'_{out} = 0.668.

quantity M'_{out} differs from the real Mach number near the lower edge of the outlet cross section because of the change in the total pressure and in the entropy on the streamlines intersecting shock waves in the local supersonic region. Of the variants of calculations given below, a special case is the last variant (y_{max} = 0.03, m = 3.3), in which the quantity M'_{out} differs from the real Mach number in the entire outlet cross section, since the shock waves are located across the flow from the lower wall of the channel to the upper wall.

To find the steady-state solution of the formulated boundary-value problem use was made of the establishment method. The unsteady solution of the initial boundary-value problem was calculated using a substantially nonoscillatory difference scheme of second order of accuracy (ENO2) that ensures the second order of accuracy everywhere, including the points of local extrema and the regions of nonsmoothness of flow [10]. We employed the modification of the ENO2 scheme proposed in [4]. The nonorthogonal, nonuniform computational grid was formed by vertical straight lines and lines obtained by subdividing the channel width into a specified number of steps. The size of the grid cells $\Delta x = \Delta y$ was constant in the central part of the channel in the region of supersonic velocities and increased in the region of subsonic flow near the inlet and outlet cross sections of the channel and in the direction of the upper wall.

In the calculations performed, the parameters of the flow approached steady-state values after $2\cdot10^4$–$1.5\cdot10^5$ time steps depending on the airfoil shape and the initial conditions. The accuracy of the method was monitored by calculating transonic flow about a circular step with a height of y_{max} for M'_{out} = 0.675 and comparing with the data available in the literature [11]. The basic calculations were performed on a 401 × 171 grid with $\Delta x = \Delta y = 0.004$ in the supersonic region. The computational experiments on a finer grid with $\Delta x = \Delta y = 0.002$ have shown that it yields no substantial increase in the degree of solvability of the shock waves; at the same time, the numerical solution obtained approached the stability boundary.

Results of Numerical Modeling. Figure 2 shows the steady-state transonic flow obtained for M'_{out} = 0.666 and the airfoil (1) with y_{max} = 0.1 and m = 2.7. The central part of the flow $1 < x < 2$, $0 < y < 0.6$ is presented on an enlarged scale in Fig. 3a. As is seen, a weak shock wave is formed at the point with coordinates of $x \approx 1.56$ and $y \approx 0.32$, which is located near the sonic line. This line ends on the airfoil and closes the first supersonic zone at

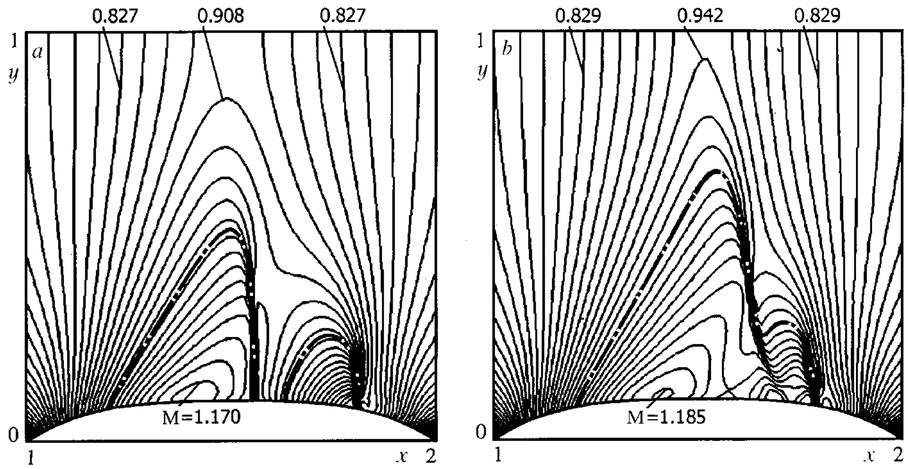

Fig. 4. Fragment of transonic flow near the airfoil. The lines of constant values of the Mach number for y_{max} = 0.1 and m = 3.3: a) M'_{out} = 0.6713; b) M'_{out} = 0.6720.

$x \approx 1.58$, so that the flow turns out to be subsonic on the portion of the airfoil $1.58 < x < 1.62$. Then the flow is accelerated again to supersonic velocities. The second zone is closed by the shock wave formed at $x \approx 1.75$, $y \approx 0.16$ according to the same scheme as the first shock wave, as a result of the coalescence of compression waves originating from the sonic line.

Figure 3b shows a fragment of the transonic flow obtained for the same airfoil and the larger Mach number M'_{out} = 0.668. As is seen, in this case the shock wave formed near the sonic line at $x \approx 1.56$, $y \approx 30$ is insufficiently strong for the supersonic region to be split into two parts. Propagating toward the airfoil, this shock wave intersects a weak shock wave formed near the center of the airfoil due to its zero curvature at x = 1.5 and then reaches the airfoil at $x \approx 1.66$. The shock wave reflected from the airfoil propagates in the direction to the sonic line and downstream. Having reached the sonic line, the reflected shock wave produces the second concave portion in the shape of a supersonic region.

A comparison of Fig. 3a and 3b shows that a qualitative change in the pattern of transonic flow is observed in the case of a very small, about 0.3%, change in the specified conventional Mach number at the outlet from the channel M'_{out}. This corresponds to an increase of 0.65 m/sec in the flow velocity (equal to 216 m/sec in this case).

The calculations of flow about the airfoil (1) with y_{max} = 0.1 and a higher m = 3.0 corresponding to the smaller curvature of the central part of the airfoil have shown that flow patterns similar to those presented in Fig. 3 are implemented for M'_{out} = 0.669 and 0.670, i.e., for a difference of 0.15% in the values of the specified Mach number.

Figure 4 shows a fragment of the transonic flow obtained for the airfoil (1) with y_{max} = 0.1 and m = 3.3. As is seen, further decrease in the curvature led to the fact that two separate supersonic zones realized for M'_{out} = 0.6713 near the airfoil coalesce into one zone, even for M'_{out} = 0.6720, i.e., with a change of 0.10% in the characteristic Mach number at the outlet.

Figure 5 shows results of calculating flow about the airfoil (1) having a smaller thickness y_{max} = 0.03. When the value of the parameter m is fixed this leads to a further decrease in the curvature of the airfoil in its central part (Fig. 1). As is clear from Fig. 5, when m = 3.3 and M'_{out} = 0.8169 there are two supersonic regions closed by the shock waves which intersect the airfoil in the normal direction. At the same time, for M'_{out} = 0.8172, i.e., with a change of 0.04% in the characteristic Mach number, we observed the coalescence of two supersonic zones; the region of supersonic flow propagated to the upper wall of the channel.

The patterns of transonic flow obtained turned out to be stable to different initial conditions and to a refinement of the grid. In employing 161 × 69 and 401 × 171 grids, the calculation results in the variants with two supersonic zones were nearly the same. At the same time, in the variants of calculations with a single supersonic wave in which oblique shock waves were present, the resolution of the shock waves was substantially higher on the fine 401 × 171 grid. The rate of establishment of the flow depended on the airfoil shape and M'_{out}. In the variants presented in Figs. 4 and 5, it was very slow so that it took about $1.5 \cdot 10^5$ time steps to obtain the steady-state solution.

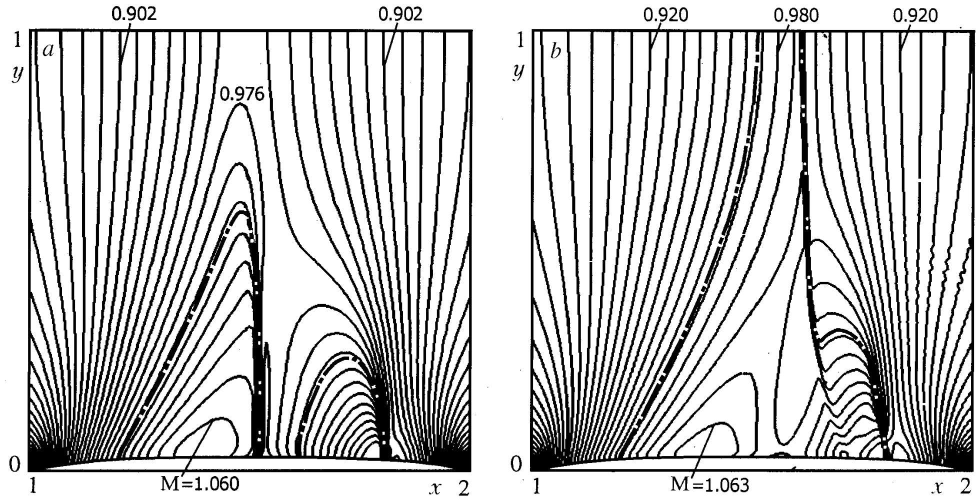

Fig. 5. Fragment of transonic flow near the airfoil. The lines of constant values of the Mach number for y_{max} = 0.03 and m = 3.3: a) M'_{out} = 0.8169; b) M'_{out} = 0.8172.

The results obtained above show that the patterns of flow about airfoils with a small relative thickness possess an extremely high sensitivity to a change in the boundary conditions. This enables us, in particular, to infer that in [5] where flow about a thin 3% airfoil was studied for the Mach number of the free flow M_∞ = 0.936, the selected 160 × 40 computational grid, apparently, did not ensure the accuracy required for analysis of the reasons for the nonuniqueness of the numerical solutions of the Euler equations.

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NOTATION

x, y, Cartesian coordinates; t, time; M_∞, Mach number of the free flow; M_d, design Mach number for the flow for which the airfoil is designed; M'_{out}, conventional Mach number at the outlet from the channel; p, pressure; p_{out}, static pressure at the outlet from the channel; m, exponent in the equation of the airfoil (1); y_{max}, maximum thickness of the airfoil; Δx and Δy, size of the grid cells in the region of supersonic flow; C_L, lift coefficient; $\rho(x, y, t)$, gas density; $u(x, y, t)$ and $v(x, y, t)$, components of the velocity vector in the direction of the x and y axes; $e(x, y, t)$, internal energy; γ, adiabatic exponent. Subscripts: out, outlet; max, maximum; d, design; L, lifting; 0, corresponds to the total pressure.

REFERENCES

1. G. B. Cosentino, *J. Aircraft*, **20**, No. 4, 377–379 (1983).
2. W. Pfenninger, J. Viken, C. S. Vemuru, and G. Volpe, *AIAA Paper*, No. 86-2625, 1–45 (1986).
3. A. N. Kraiko and K. S. P'yankov, *Zh. Vych. Mat. Mat. Fiz.*, **40**, No. 12, 1890–1904 (2000).
4. A. I. Kotov and A. G. Kuz'min, *Vestn. SPbGU*, Ser. 1, Issue 3, 87–91 (2000).
5. K. McGrattan, *AIAA J.*, **30**, No. 9, 2340–2343 (1992).
6. M. M. Hafez and W. H. Guo, *Comput. Fluids*, **28**, Nos. 4–5, 701–719 (1999).
7. A. G. Kuz'min, *Boundary Value Problems for Transonic Flow*, Wiley, Chichester (2002).
8. C. A. J. Fletcher, *Computational Techniques for Fluid Dynamics* [Russian translation], Moscow (1990).
9. S. Edelman, P. Colella, and R. P. Shreeve, *AIAA Paper*, No. 83-1941, 1–9 (1983).
10. J. Y. Yang and C. A. Hsu, *AIAA J.*, **30**, 1570–1575 (1992).
11. R. H. Ni, *AIAA J.*, **20**, 1565–1571 (1982).